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Barriers to Entailment

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Back to Hume's Law

— So now we want to see whether we can do the same thing in the controversial case: the *is/ought* barrier.
— Like with the other cases, we need a logic to get started.
— A good place to start is with *deontic* logic.

It's having a logic that gives us a determinate enough sense of "get" that we can potentially prove that you can't "get an *ought* from an *is*."

A Simple Deontic Logic

Where modal logic is the logic of necessity and possibility (*must* and *might*), denotic logic is the logic of *ought* and *permissibility* (*should/ought*, and *is permitted to/may*.)

sentence letters:	p, q, r, p_1, p_2 etc.
logical connectives:	$\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$
deontic operators:	O, P
punctuation:	(,)

1. If ϕ is a sentence letter, then ϕ is a sentence.
2. If ϕ and ψ are sentences, then
   a. $\neg\phi$ is a sentence.
   b. $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$, $(\phi \leftrightarrow \psi)$ are sentences.
   c. $O\phi$ and $P\phi$ are sentences.
3. Nothing else is a sentence.

— This has two main operators: O and P, that can informally be read as *it ought to be the case that* and *it is permissible that*.
— So Op means it ought to be the case that p, and Pp means it is permissible that p.

Models

A DL-model a quadruple $\langle W, S, @, I \rangle$, where

— W is a non-empty set of points (the set of *possible worlds*),
— S is a non-empty subset of W (the set of *superb worlds*)
— @ is an element of W (the 'actual world')
— I is an interpretation function mapping pairs of sentence letters and elements of W into the set of values $\{1, 0\}$.

← Think of S as the set of worlds where things are superb/permissible/really great in virtue of meeting the normative standards.

The deontic operators have the following truth-conditions:

$$V_M(\mathcal{P}\phi, w) = 1 \text{ iff there is } u \in S \text{ such that } V_M(\phi, u) = 1$$
$$V_M(\mathcal{O}\phi, w) = 1 \text{ iff for all } u \in S,\ V_M(\phi, u) = 1$$

– Truth in the model is truth at the model's actual world.
– $\Gamma \vDash_{DL} \phi$ iff all DL-models of Γ are models of ϕ.

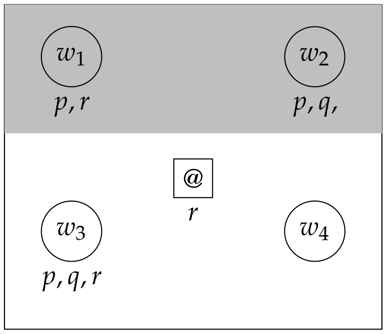


Figure 1: A diagram depicting a DML Model. The shaded area represents the set of superb worlds.

What could our Inter-model relation be?

What changes are normative sentences *affected* by?

particular/universal	extension
past/future	future-switching
is/*must*	modal-extension
non-indexical/indexical	context-switching
descriptive/normative	???

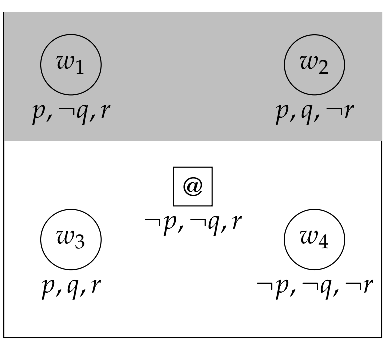
— Informal (in Humberstone and attributed to Toomas Karmo):

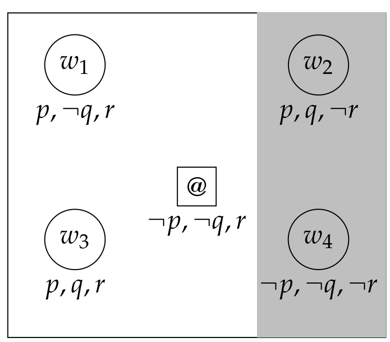
normative claims are sensitive to *changes in the normative standards*.[1]

[1] Karmo, T. (1988). Some valid (but no sound) arguments trivially span the 'is'-'ought' gap. *Mind*, 97(386):252–257

— What does that mean? What does "changing normative standards" look like in DML-models?

— It looks like changing which (non-empty) subset of W is "S". Call this *S-shifting*.





The model on the right is just like the one on the left except for the shaded area (representing the subset S.) It is an S-shift of the original model.

Some problems:

— Peter Vranas: it's a mistake to classify $p \vee Op$ as Neither (descriptive nor normative).[2]
— Gerhard Schurz: it's a mistake to think you've proved Hume's law if you've proved something about a logic that does not have the alethic modal operators $\Box$ and $\Diamond$ in, in addition to O and P. What about counterexamples like:

$$\Box p \vDash Op$$
$$\neg\Diamond p \vDash \neg Op$$

— Op isn't s-shift fragile!

[2] Vranas, P. (2010). Comment on 'Barriers to Implication'. In Pigden, C., editor, *Hume, Is and Ought: New Essays*, pages 260–267. Palgrave MacMillan


Peter Vranas

Well, let's start with the easy stuff. We can add the modal operators $\Box$ and $\Diamond$ to our language.

$V_M(\Diamond\phi, w) = 1$ iff there is $u \in W$ such that $V_M(\phi, u) = 1$
$V_M(\Box\phi, w) = 1$ iff for all $u \in W$, $V_M(\phi, u) = 1$

The problem is that the language is not expressive enough to formulate the counterexamples. So it might be true that no descriptive DL sentences entail any normative DL sentences, but only because the language isn't rich enough to express the counterexamples.

Complex Counterexamples

OK, what are we going to do about this:

$$\neg\Diamond p \vDash \neg Op$$


Gerhard Schurz

Some possible responses

1. say that $\neg\Diamond p$ is really normative
2. say that $\neg Op$ is not really normative
3. say that the argument isn't valid ([3])
4. restrict the barrier ([4])

[3] Pigden, C. R. (1989). Logic and the autonomy of ethics. *The Australasian Journal of Philosophy*, 67:127–151
[4] Schurz, G. (1991). How far can Hume's is-ought thesis be generalised? : An investigation in alethic-deontic modal predicate logic. *Journal of Philosophical Logic*, 20:37–95

Friends of *Ought Implies Can*:

(DM1) $O\phi \vDash \Diamond\phi$
(DM2) $\neg\Diamond\phi \vDash \neg O\phi$
(DM3) $\Box\phi \vDash P\phi$
(DM4) $\neg P\phi \vDash \neg\Box\phi$
(DM5) $P\phi \vDash \Diamond\phi$
(DM6) $\neg\Diamond\phi \vDash \neg P\phi$
(DM7) $\Box\phi \vDash O\phi$
(DM8) $\neg O\phi \vDash \neg\Box\phi$

Will implies Can

Only possible stuff can happen in the future.

$$F\phi \vDash \Diamond\phi$$

$$\neg\Diamond\phi \vDash \neg F\phi$$

Again we find a similar range of corollaries:

(TM1) $G\phi \vDash \Diamond\phi$
(TM2) $\neg\Diamond\phi \vDash \neg G\phi$
(TM3) $\Box\phi \vDash F\phi$
(TM4) $\neg F\phi \vDash \neg\Box\phi$
(TM5) $F\phi \vDash \Diamond\phi$
(TM6) $\neg\Diamond\phi \vDash \neg F\phi$
(TM7) $\Box\phi \vDash G\phi$
(TM8) $\neg G\phi \vDash \neg\Box\phi$

Another new logic:

TML = tense modal logic, DML = deontic modal logic

A TML model is an ordered 5-tuple,

$$(W, T, @, n, I)$$

where:

1. W is a non-empty set of points (the set of worlds)
2. T is a non-empty set of integers (the set of times)
3. $@ \in W$ (the model's *actual* world)
4. $n \in T$ (the model's *now*)
5. I assigns each sentence letter an intension: a function from $T \times W$ into $\{1, 0\}$.

An example of a TML-model:

	w	**@**	**u**
t_4	p, q	p, q, r	p
t_3	p, q	p, q	p
n	p, r	p	p
t_2	p	p	p, q
t_1	p, q	p, q	p, q

Some key details:

1. $V_M(F\phi, t, w) = 1$ iff there is $u \in T$ s.t. $t < u$ & $V_M(\phi, u, w) = 1$
2. $V_M(\Diamond\phi, t, w) = 1$ iff there is $u \in W$, $t^* \in T$ s.t. $V_M(\phi, t^*, u) = 1$
3. $V_M(\Box\phi, t, w) = 1$ iff for all $u \in W$, $t^* \in T$, $V_M(\phi, t^*, u) = 1$
4. A sentence is true in the model if it is true at $(n, @)$.
5. $\Gamma \vDash \phi$ just in case all models that make Γ true make ϕ true.
6. This logic validates all the TM-principles.

(TM5) $\mathcal{F}\phi \vDash \Diamond\phi$

Proof. Suppose that $V_M(\mathcal{F}\phi) = 1$. Then $V_M(\mathcal{F}\phi, n, @) = 1$ and there is $t^* \in T$ such that $n < t^*$ and $V_M(\phi, t^*, @) = 1$. So $V_M(\Diamond\phi, n, @) = 1$ and $V_M(\Diamond\phi) = 1$. $\square$

Restricted and unrestricted future-switching

Unrestricted Future-switching

	w	**@**	**u**
t_5	p, q	p, q, r	p
t_4	p, q	p, q	p
n	p, r	p	p
t_2	p	p	p, q
t_1	p, q	p, q	p, q

	w	**@**	**u**
t_5	p, q		p
t_4	p, q		p
n	p, r	p	p
t_2	p	p	p, q
t_1	p, q	p, q	p, q

Figure 2: Unrestricted Future-switching. The grey box represents the model's *actual future*.

— This would make Fp and Gp fragile.
— But it will make $\Box p$ fragile too!

Restricted Future-switching

	w	**@**	**u**
t_5	p, q	p, q, r	p
t_4	p, q	p, q	p
n	p, r	p	p
t_2	p	p	p, q
t_1	p, q	p, q	p, q

	w	**@**	**u**
t_5	p, q	p	p, q, r
t_4	p, q	p	p, q
n	p, r	p	p
t_2	p	p	p, q
t_1	p, q	p, q	p, q

Figure 3: Restricted Future-switching

— If this is what counts as future-switching, then $\Box p$ will not count as future. Great!
— Problem: $\mathcal{F}p$ and $\mathcal{G}p$ are not fragile w.r.t. this relation. For if $\Box p$ is true in a model, there will be no restricted future-switching that makes p false at future times.
— Saying that $\Box p$ was Future—as we would on option 1—was a bit odd. Saying that $\mathcal{F}p$ and $\mathcal{G}p$ are not Future—as we would on option 2—is, I think, completely intolerable.

Question for tomorrow:

What if we kept restricted future-switching, but instead of fragility, we considered *breakability* with respect to the relation?

Why it is promising:

— Fp and Gp are restricted-future-switch breakable.
— $\Box p$ is not.
— $p \to \Box q$ is also restricted future-switch breakable.

Why it is less promising:

— We haven't proved anything about breakability (only fragility) and we know $\neg p \vDash p \to \Box p$

References

Karmo, T. (1988). Some valid (but no sound) arguments trivially span the 'is'-'ought' gap. *Mind*, 97(386):252–257.

Pigden, C. R. (1989). Logic and the autonomy of ethics. *The Australasian Journal of Philosophy*, 67:127–151.

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