

# Foundations Seminar: Barriers to Entailment

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Week 3: first steps towards a proof

- This week we'll look at three earlier attempts to prove Hume's Law.
  1. Charles Pigden's approach is a precisification of a principle he calls the *conservativeness of logic*. He proves it about first-order classical logic, and denies that he needs to prove it about any more expressive logic.
  2. Gerhard Schurz proves his about a family of complex modal-deontic-predicate logics. He emphasises the importance of working in richer logics.
  3. R&R take a model-theoretic approach and attempt to prove 4 of our 5 barriers to entailment. They work with modal, tense, and denotical logics, but do not attempt to combine these into more complex logics.
- Our goal today is to get a handle on how each of the proofs works, how they differ, and what some of their strengths and weaknesses are.

R&R is "Restall and Russell"—I'm just going to refer to us as "R&R" (so kind of in the third person, plural) when talking about the older version of my view. Is that the best approach? I don't know, but it comes fairly naturally in this context.

## Six Tests for a formulation of Hume's Law

1. Can it get past Prior's dilemma?

$$\frac{p}{p \vee Oq} \qquad \frac{p \vee Oq}{\neg p} \qquad \frac{\neg p}{Oq}$$

2. Classification of Vranas sentences?

$$p \rightarrow Oq, \quad p \vee Oq$$

Both Schurz and Vranas argue that it is important to classify these as normative.<sup>1</sup>

"This escape from Prior's paradox by excluding mixed sentences is not a truly satisfying solution. For in all ethical and juridical theories mixed sentences play an important role—think of conditional obligations, represented by sentences of the form  $\Box(p \rightarrow Oq)$  or  $\forall x\Box(Fx \rightarrow OGx)$ . A satisfying solution of Prior's paradox must show what Hume's general philosophical principle of the is-ought dichotomy implies for deductions containing mixed sentences. In other words, we need something which we will call the general Hume thesis, abbreviated GH—that is a natural generalization of Hume's thesis for deductions containing also mixed sentences."<sup>2</sup>

<sup>1</sup> Vranas, P. (2010). Comment on 'Barriers to Implication'. In Pigden, C., editor, *Hume, Is and Ought: New Essays*, pages 260–267. Palgrave MacMillan

<sup>2</sup> Schurz, G. (2010). Non-trivial versions of Hume's is-ought thesis. In Pigden, C., editor, *Hume, Is and Ought: New Essays*, pages 198–216. Palgrave MacMillan

3. *Ought* implies *can* (complex contraposition)

$$Op \models \Diamond p$$

$$\neg \Diamond p \models \neg Op$$

"Many ethical philosophers appear to accept the view that 'ought' implies 'can'. This view, which seems quite plausible, can perhaps be formulated more precisely as (1) Statements of the form 'N ought to do X' entail the corresponding statements of the form 'N can (is able to) do X.' But (1) is equivalent to (2) Statements of the form 'N cannot (is unable to) do X' entail the corresponding statements of the form 'It is not the case that N ought to do X'. And (2) appears to say that there is a non-normative statement which entails a normative one." (Mavrodes, 1964, 42)

## 4. Informal arguments (speech acts, quotation, thick normative expressions)

We talked a lot about these last week.

## 5. demarkation of normative/descriptive

Does the account differentiate cleanly between sentences in the premise class (i.e. descriptive sentences) and those in the conclusion class (i.e. normative sentences)? Is there still space for a stalemate here?

## 6. unified treatment of the barriers. The barriers have a great deal in common. Does the account help us see why? Does it treat them the same way (or explain why it does not)? Can it be generalised to the others?

These are not all the critical questions we can ask about an approach, but they constitute a manageable task for today.

*Pigden*

- Pigden knows a lot about the history of ethics and his work is often helpful for situating the debate over Hume's Law within a broader historical context
- His paper distinguishes three forms of autonomy:
  1. ontological autonomy:
    - "the thesis that moral judgements, to be true, must answer to a realm of *sui generis* non-natural properties."
    - Can be understood as the reverse of naturalism about ethics

"(Con)" for the (Con)servativeness of Logic.

- Pigden thinks only the ontological thesis in tension with naturalism.
- 2. semantic autonomy:
  - "a realm of *sui generis* non-natural predicates, which do not mean the same as any natural counterparts."
  - Pace G.E. Moore not in tension with naturalism because the identities between natural and non-natural predicates may be synthetic.
- 3. logical autonomy:
  - "maintains that moral conclusions *cannot be derived* from non-moral premises." (28)<sup>3</sup>
  - Also compatible with naturalism.
- Pigden aims only to defend logical autonomy
 

"I argue that logical autonomy—from now on, unless otherwise stated, *the* autonomy of ethics—is merely an instance of the more general theory that logic is conservative." (129)

"Why does [younger] Prior think that ethics is autonomous? His argument is simple, and depends upon the rather shaky principle that logic preserves, but does not extend, truth and content. You can't get out of a valid inference what you haven't put in." (129)
- Before precisification this is more an intuitive principle than a theorem.
- Goal will be to make that precise enough to be provable.
- Will give us a version of Hume's Law on which ethics isn't special.
- Sure, if you didn't use the word *ought* in the premises, you can't have it in the conclusion.
- But if you didn't use the word *hedgehog* in the premises, you can't get it out in the conclusion.
- Because of this Pigden doesn't actually need to define what a normative sentence is—even if *hedgehog* turned out to be a normative word, he thinks the autonomy of ethics would hold.
- He calls his interpretation of Hume's law "demonstrable but unexciting" (129)

OK, how can we make the intuitive idea of the conservativeness of logic more precise?

- In general it is not true that expressions that don't appear in the premises of a valid argument can't appear in the conclusion.
- Take for example, logical expressions:
  1.  $p \models \neg\neg p$
  2.  $\neg\exists x\neg Fx \models \forall x Fx$

This is an important idea to get your head around. The identity between e.g. *good* and *causes happiness for the greatest number* might be like the identity between *Hesperus* and *Phosphorus*—synthetic, i.e. true, but not true in virtue of the meanings of the expressions alone.

<sup>3</sup> Pigden, C. R. (1989). Logic and the autonomy of ethics. *The Australasian Journal of Philosophy*, 67:127–151

Q. What consequences does this have for the informal counterexamples?

Q: Is it plausible that Hume was only expressing the logical thesis?

"If ethics is not unique in its autonomy, what has so much been made of it? [sic] One reason is that truisms can acquire importance by being denied." (146)

- 3.  $\models a = a$
- Or arguments we've considered already:
  - 1.  $p \models p \vee q$
  - 2.  $p, \neg p \models q$
  - 3.  $p \models q \vee \neg q$
- So the principle can't say "no expressions not in the premises of a valid argument appear in the conclusion." Here is what it does say:  
(Con) A predicate or propositional variable cannot occur non-vacuously in the conclusion of a valid argument unless it appears among the premises. (136)
- an expression occurs vacuously in the conclusion of an argument just in case uniformly substituting another expression of the same syntactic type preserves validity.
- So we have two restrictions: to predicates and propositional variables.<sup>4</sup> And to non-vacuous occurrences.
- (Prior 1) fails to be a counterexample because the new normative disjunct occurs vacuously.
- BUT (Con) has to be restricted to exclude logical expressions (hence the restriction to "predicate or propositional variables") otherwise it would be in conflict with the following:
- So what about *normative* logical expressions, as might feature in arguments like:
  - 4.  $\models Op \rightarrow Pp$
  - 5.  $\neg \Diamond p \models \neg Op$
- But it turns out that Pigden denies that there *are* any normative logical expressions. So he denies that these arguments are really valid. On his view only first-order predicate logic is logic proper.

See (Pigden, 1989) (136) for further explanation, illustration, and equivalent formulations.

<sup>4</sup> Should really be non-logical expressions, otherwise we have trouble with the predicate =.

## Schurz

Gerhard Schurz has long maintained in the *is/ought* case:

"[A]n adequate treatment of the problem requires that the modal language contains the alethic necessary operator  $\Box$  as well as the deontic obligation operator  $O$ —i.e. we have to work within an alethic and modal predicate language."<sup>5</sup> (37)

Some definitions:

- An **elementary normative** sentence is of the form  $O\phi$ .
- **Purely descriptive** sentences contain no occurrence of  $O$ .
- **Purely normative** ones are built up from **elementary normative** sentences with logical operators.

<sup>5</sup> Schurz, G. (1991). How far can Hume's is-ought thesis be generalised? : An investigation in alethic-deontic modal predicate logic. *Journal of Philosophical Logic*, 20:37–95  
(GH) is for Generalised Hume Thesis.

- Sentences which are neither purely normative nor purely descriptive—  
e.g.  $p \vee Oq$ ,  $p \rightarrow Oq$ —are **mixed**.

(GH) Every deduction  $\Gamma \vdash \phi$ , where the members of  $\Gamma$  are *purely descriptive* and the conclusion  $\phi$  is either *purely normative* or *mixed*, is such that  $\phi$  is an O-irrelevant conclusion of  $\Gamma$ .

- $A$  is an O-irrelevant conclusion of  $\Gamma$  just in case each sentence letter in the conclusion which is in the scope of an O-operator could be replaced with any other sentence *salva validitate*.
- (GH) is an example of a theorem with an exception; we might rephrase:  $\Gamma \not\vdash \phi$  *unless*  $\phi$  is an O-irrelevant conclusion of  $\Gamma$ .
- It is this last feature that allows the theorem to respond to the famous counterexample from Prior:

$$p \models p \vee Oq \quad (\text{Prior 1})$$

Prior, A. N. (1960). The autonomy of ethics. *The Australasian Journal of Philosophy*, 38:199–206

The  $q$  in the conclusion could be any sentence at all, so it is O-irrelevant, so this argument is not a counterexample to (GH)—even though  $p$  is descriptive and  $p \vee Oq$  is *mixed*.

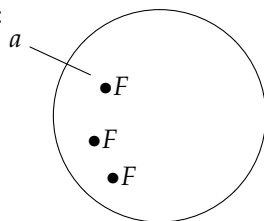
- Distinguishing the normative from descriptive sentences: Schurz writes “in our logical framework we must presuppose that under the primitive symbols we can separate those with normative (or valiative) meanings. E.g. ‘it is necessary’, ‘it is desired’, etc. have descriptive meaning, whereas ‘it is obligated’, ‘it is a value’ have ethical meaning. Without this separation we would not even be able to *formulate* the is-ought problem.” (9) <sup>6</sup>

<sup>6</sup> Schurz, G. (1997). *The Is/Ought Problem: An Investigation in Philosophical Logic*. Kluwer, Dordrecht

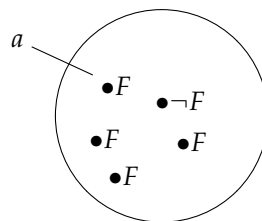
### 3. R&R

- The theorem in (Restall and Russell, 2010) works differently. It aims at proving Hume’s Law as just one of a group of philosophical “barriers to entailment”  $\Rightarrow$
- it is proved about logics defined over classes of models.
- Particular vs Universal

- Our first task is to say what it is for sentences to be *particular* and *universal*.
- Central idea: True particular claims stay true when the model is extended:

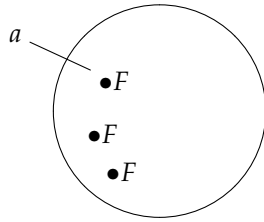


$Fa$  is true

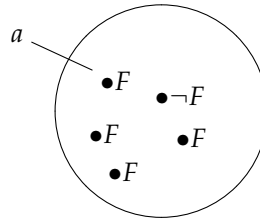


$Fa$  is still true

- But true Universal claims can be made false by extending the model:



$\forall xFx$  is true



$\forall xFx$  is no longer true

- The premise (e.g. descriptive) and conclusion (e.g. normative) classes are defined as containing sentences which are, respectively, *preserved* or *fragile* over certain relations,  $R$ , on the models.
- In the particular/universal case that relation is model-extension.

**Definition 1 (Model-Extension ( $\subseteq$ ))** A model  $N$  extends a model  $M$  just in case

1.  $D_M \subseteq D_N$  and
2. for all individual constants  $\alpha$ ,

$$I_N(\alpha) = I_M(\alpha)$$

3. for all  $n$ -place predicates  $\Pi$ ,

$$I_M(\Pi) \subseteq I_N(\Pi)$$

**Definition 2 (R-preserved and -Fragile)** A sentence is

- R-preserved iff any model which satisfies it is such that all R-related models satisfy it too.
- R-fragile iff all models which satisfy it are R-related to at least one model which does not.

**Definition 3 (Particular and Universal Sentences)** A sentence is

- Particular iff it is  $\subseteq$ -preserved.
- Universal iff it is  $\subseteq$ -fragile.

- Let's try to understand these definitions better by looking at some examples.

Sentences which are  $\subseteq$ -preserved (Particular):

$$Fa, \neg Fa, Fa \vee Gb, Fa \rightarrow Gb, \exists xFx, \neg \forall xFx, \forall x(Fx \vee \neg Fx)$$

Sentences which are  $\subseteq$ -fragile (Universal) :

$$\forall xFx, \neg \exists x \neg Fx, \forall x \forall y Rxy, \forall x(Fx \wedge Gx)$$

Sentences which are neither Particular nor Universal:

$$Fa \vee \forall xGx, \quad \neg Fa \rightarrow \forall xGx$$

Sentences which are Both Particular and Universal (!):

$$Fa \wedge \neg Fa$$

Then the theorem says:

(R&R) No satisfiable set of R-preserved sentences entails an R-fragile sentence

This turns out to be easy to prove!

**Proof 1** Suppose the set of premises  $\Gamma$  is satisfiable and the conclusion  $\phi$  is R-preserved. Then  $\Gamma$  has a model. Does that model make  $\phi$  true? If not, we have a counterexample. If it does, that model has an R-related model which makes the  $\phi$  false (from the definition of R-fragile) and that very model satisfies  $\Gamma$  (from the definition of R-preserved). So either way there is a model that makes  $\Gamma$  true and  $\phi$  false, so  $\Gamma \not\models \phi$ .

— What will this say about the Universal analogue of Priori (1)?

$$Fa \models Fa \vee \forall xGx \quad (1)$$

— The descriptive/normative instance of (R&R) is more complicated in ways that aren't essential here, but the response to Prior is parallel.  $p$  is descriptive but  $p \vee Oq$  is not normative, and so the argument is not a counterexample to (R&R).

## Comparing Results

Test	Pigden	Schurz	R&R
Prior's Dilemma			
Classification of Vranas sentences			
<i>Ought</i> implies <i>Can</i>			
informal (Searle, thick n.e., quotation etc.)			
demarkation of normative/descriptive			
unified treatment of barriers?			

## References

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